

# Calculus 1

Final Exam

November 4, 2022 (8:30 – 10:30)



university of  
 groningen

**Please read the instructions!**

1) Prove using the  $\varepsilon$ - $\delta$  definition that  $\lim_{x \rightarrow 1} \frac{3x^2 - 3}{x - 1} = 6$ .

2) Apply l'Hospital's Rule to find the following limit:  $\lim_{h \rightarrow 0} \frac{(1+h)^{1/h} - e}{h}$ .

3) We say that a function  $f$  has a *fixed point* at  $a \in \text{dom}(f)$  if  $f(a) = a$ . Prove the following statement. [Hint: Use the Mean Value Theorem.]

“If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is differentiable everywhere and  $f'(x) \neq 1$  for all  $x \in \mathbb{R}$ , then  $f$  has at most one fixed point.”

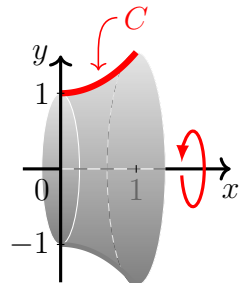
4) Use the Maclaurin series for  $f(x) = (1+x^2) \ln(1+x)$  to find  $f^{(2022)}(0)$ .

5) Prove, via mathematical induction on  $N$ , that

$$\int_0^{\infty} x^N e^{-x} dx = N! \quad \text{for every integer } N \geq 0.$$

(Recall that  $0! = 1! = 1$  and  $n! = n \cdot (n-1) \cdots 2 \cdot 1$  for  $n = 2, 3, \dots$ )

6) Determine the surface area of the solid obtained by rotating the curve  $C = \{(x, \cosh x) \mid 0 \leq x \leq 1\}$  about the  $x$ -axis. (See the figure.) Include all sides!



7) Evaluate the definite integral  $\int_2^{2\sqrt{3}} \frac{x+1}{x\sqrt{16-x^2}} dx$ .

8) Solve the initial value problem  $y'(x) - \frac{y(x)}{x} = x^2 + 3x - 2$ ,  $y(1) = 4$ .

9) Find all complex number solutions of the equation  $z^2 = \bar{z}$ . Write your final answer in algebraic form.

# Instructions

- **write your name and student number on the envelope and on the top of each sheet of writing paper!**
- use the writing (lined) and scratch (blank) paper provided, raise your hand if you need more paper
- start each question on a new page
- use a pen with black or blue ink
- do not use any kind of correcting fluid or tape
- any rough work should be crossed through neatly so it can be seen
- this exam is open-book, you may use the textbook or the lecture notes
- you are allowed to use a simple pocket calculator
- programmable/graphing calculators are not allowed, nor the use of electronic devices (tablet, laptop, phone, etc.) to solve the exercises
- your work should be clearly and logically structured
- **explain your reasoning using words**
- show all your calculations, an answer without any computation will not be rewarded
- each problem is worth 10 points
- upon completion<sup>1</sup> place your worksheets in the envelope and submit them at the front desk

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<sup>1</sup>At the end of the exam or after you finished whichever is sooner.